Unit - 2

Number System

CPU of a computer can understand information which composed of zero's and One's.

Number systems are used to represent the quantity.

There are four number systems which are given below.

1. Decimal number system
2. Binary number system
3. Octal number system
4. Hexadecimal number system.

These number systems are widely used in microprocessor and computers.

Decimal Number System

The decimal number system contain ten distinct symbols that are 0,1,2,3,4,5,6,7,8,

The base or radix of a number system

↳ The number of different symbols which can occur at each position in a number system.

→ Decimal number system is a weighted number system. i.e. for finding the

multiplied by the base raised to power depending upon the digit position.

For ex

3 2 1 0 ← Position

Number = 3642

$$3 \times 10^3 + 6 \times 10^2 + 4 \times 10^1 + 2 \times 10^0$$

Binary Number System

↳ It uses only two distinct symbols that are 0 and 1.

The left most bit is known as Most Significant bit (MSB) and right most bit is known as Least Significant Bit (LSB) in binary number system.

For ex :-

1 0 0 1 1

MSB ↵ ↳ LSB

Bit → A single binary digit is known as bit.

→ Base of Binary Number System is 2 and it is also Weighted number system

Octal Number System

In This system, there are 8 distinct

symbols to represent octal number i.e. 0, 1, 2, 3, 4, 5, 6, 7

For ex:- $(736)_8$ (2 1 0)

$= 7\times8^2 + 3\times8^1 + 6\times8^0$

Hexadecimal Number System

This number system is most widely used in computer.

There are 16 distinct symbols which represent hexadecimal numbers. So, the base of hexadecimal number system is 16. ie. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F.

This system is called alphanumeric system because the numeric digits and alphabets are used to represent it.

For ex:- $(A37.15)$ (2 1 0 -1 -2)

Conversion

Decimal →
- Binary
- Octal
- Hexadecimal

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(1) Decimal to binary

$(39.15)_{10} \rightarrow (?)_2$

2 | 39		1
	19	1
	9	1
	4	0
	2	0
	1 ⟶	1

continuous divisi
of integer numb

continuous multiply fraction part

$0.15 \times 2 = 0.30$	0
$0.30 \times 2 = 0.60$	0
$0.60 \times 2 = 1.20$	1

$(39.15)_{10} = (100111.001)_2$

Binary to Decimal

$(1011.01)_2 \rightarrow (?)_{10}$

$3\ 2\ 1\ 0\ -1\ -2$

$(1\ 0\ 1\ 1\ .\ 0\ 1)_2$

$= 1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0 + 0\times2^{-1} + 1\times2^{-2}$

$= 8 + 0 + 2 + 1 + 0 + \frac{1}{2^2}$

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$= 11 + \frac{1}{4}$

$= 11 + 0.25$

$= (11.25)_{10}$

Decimal to octal

$(127.20)_{10} \rightarrow (?)_8$

8	127	7
	15	7
	1	1

$0.20 \times 8 = 1.60$ — 1

$0.60 \times 8 = 4.80$ — 4

$0.80 \times 8 = 6.40$ — 6

$(127.20)_{10} = (177.146)_8$

Octal to decimal

$(127.10)_8 \rightarrow (?)_{10}$

$$(127.10)_8$$

$$= 1\times8^2 + 2\times8^1 + 7\times8^0 + 1\times8^{-1} + 0\times8^{-2}$$

$$= 64 + 16 + 7 + \frac{1}{8} + 0$$

$$= 87 + \frac{1}{8}$$

$$= 87 + 0.125$$

$$(127.10)_8 = (87.125)_{10}$$

Decimal to Hexadecimal

$$(193.20)_{10} \rightarrow (?)_{16}$$

16 | 193

12 → 12 (C)

1

$0.20 \times 16 = 3.20$ → 3

$0.20 \times 16 = 3.20$ → 3

$(193.20)_{10} = (C1.33)_{16}$

Hexadecimal to decimal

$(A2B.510)_{16} \longrightarrow (\quad)_{10}$

$$\overset{2\,1\;\;0\;\;-1\;-2\;-3}{(A\,2\,B\cdot 5\,1\,0)}$$

$$10\times16^{2} + 2\times16^{1} + 11\times16^{0} + 5\times16^{-1} + 1\times16^{-2} + 0\times16^{-3}$$

$$2560 + 32 + 11 + \frac{5}{16} + \frac{1}{256}$$

$$= 2603 + 0.31 +$$

$$=$$

Binary
- octal
- Hexadecimal

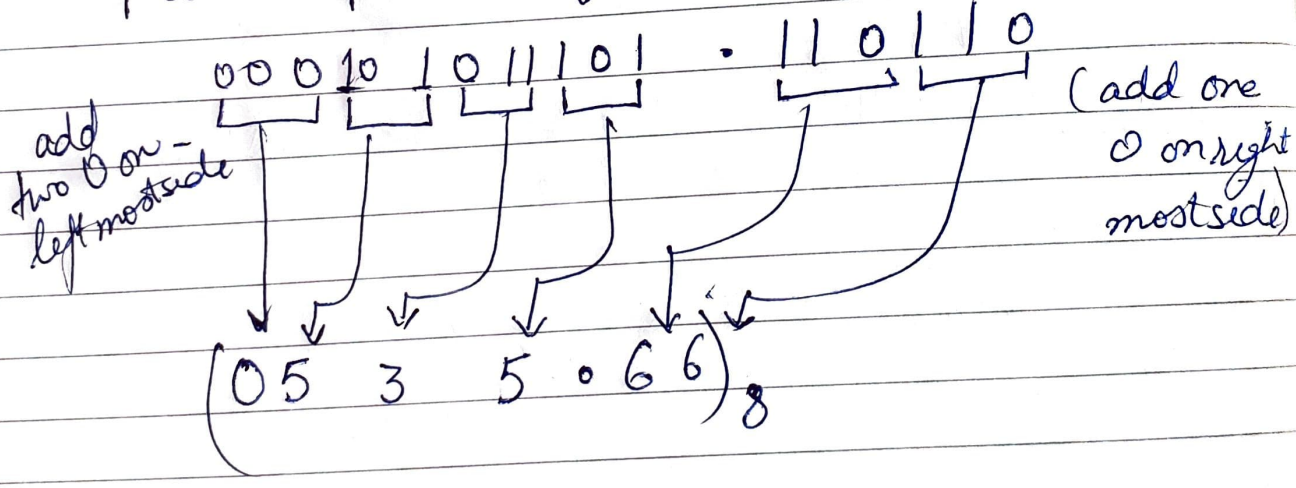
Binary to Octal

Octal number	Binary Equivalent (2^3 2^2 2^1)
0	0 0 0
1	0 0 1
2	0 1 0
3	0 1 1
4	1 0 0
5	1 0 1
6	1 1 0
7	1 1 1

Convert

$(010\,1011101 \cdot 11011)_2 = (?)_8$

Make pair of 3 bits



$(010\,1011101 \cdot 11011)_2$ $(0535.66)_8$

octal to binary

$(736.15)_8 \rightarrow (\quad)_2$

111 011 110 . 001 101

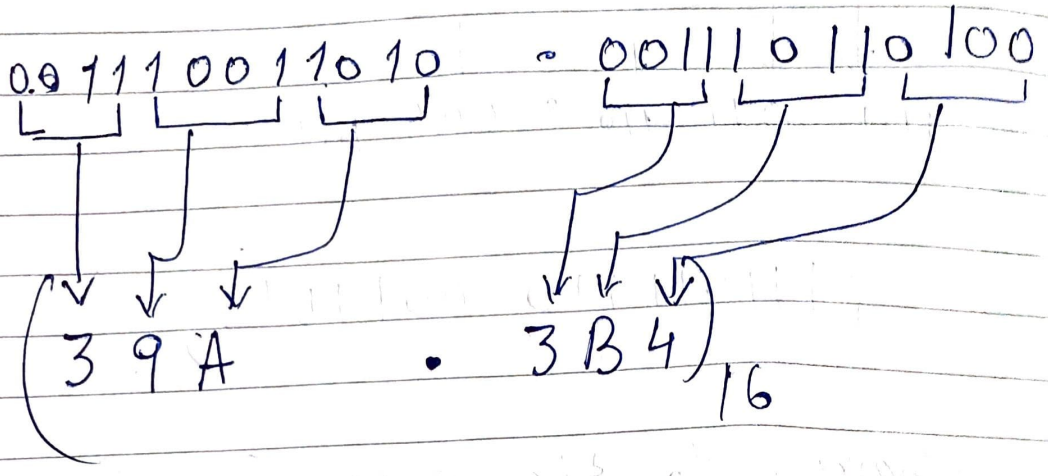
$(111011110.001101)_2$

Binary to Hexadecimal

Hexadecimal number	Binary equivalent $2^3\ 2^2\ 2^1\ 2^0$
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 0 1 1
4	0 1 0 0
5	0 1 0 1
6	0 1 1 0
7	0 1 1 1
8	1 0 0 0
9	1 0 0 1
10 = A	1 0 1 0
11 = B	1 0 1 1
12 = C	1 1 0 0
13 = D	1 1 0 1
14 = E	1 1 1 0
15 = F	1 1 1 1

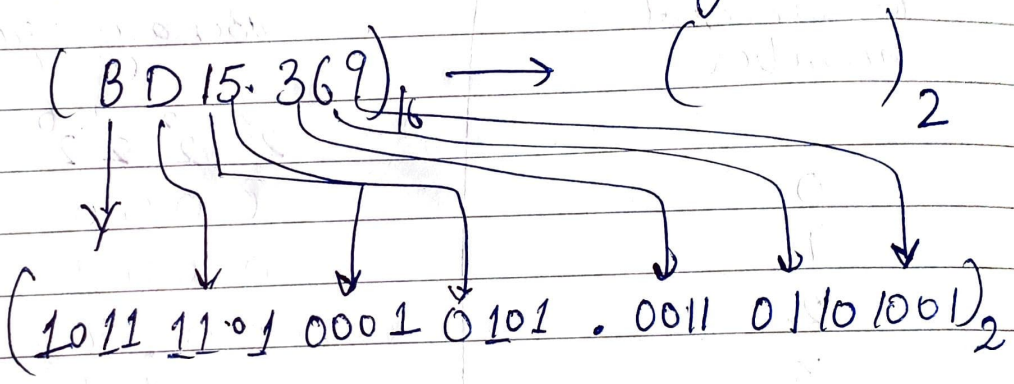
$(1110011010.00111010100)_2$

Make pair of 4 bits



Hexadecimal to binary

$(BD15.369)_{16} \rightarrow (\quad)_2$



Octal to Hexadecimal

$(735.15)_8 \rightarrow (\quad)_{16}$

Method (Any One)

- Octal → Decimal → Hexadecimal
- Octal → Binary → Hexadecimal

$(735.15)_8 = (?)_2 = (?)_{16}$

7 35 . 15

$(111\ 011\ 101 . 001\ 101)$

Now make groups of 4bit

$0001\ 1101\ 1101 . 0011\ 0100$

$(1DD.34)_{16}$

Hexadecimal to octal

$(7BF.A35)_{16} \longrightarrow (?)_8$

Method

Hexadecimal → Decimal → Octal

Hexadecimal → Binary → octal

7 B F . A 35

$(0111\ 1011\ 1111 . 1010\ 0011\ 0101)_2$

$(0\,111\;1011\;1111 \cdot 1010\;0011\;0101)_2$

$(3677.5065)_8$

Binary Arithmetic

Such like arithmetic operations, same operations can be performed on binary numbers because the design of digital circuits are very easy by using binary numbers as compared to decimal numbers.

These binary operations are;-

1. Binary ~~Addition~~
2. Binary Subtraction
3. Binary Multiplication
4. Binary Division

Binary Addition

Rules;

$0+0=0$

$0+1=1$

$1+0=1$

$1+1=0$ with carry 1.

If $1+1=1$ with carry 1

Example

```
  (1)(1)
   1 1 1 1
+  0 1 1 0
-----------
  1 0 1 0 1
```

Ans

Binary Subtraction

Rules

0 - 0 = 0

0 - 1 = 1 (with a borrow from the next column of minuend)

1 - 0 = 1

1 - 1 = 0

10 - 1 = 1

Example

```
  2 2 2
  1 0 0 0
-   0 0 1 1
-----------
  1 0 0 1
```

Ans.

Binary Multiplication

Rules

0 X 0 = 0

0 X 1 = 0

1 X 0 = 0

1 X 1 = 1

Ex:-

$$
\begin{array}{r}
1\,0\,1\,0\,1 \\
\times\ 1\,1 \\
\hline
1\,0\,1\,0\,1 \\
1\,0\,1\,0\,1\,\times \\
\hline
1\,1\,1\,1\,1\,1
\end{array}
$$

Ans

Binary Division

Rules

$0 \div 1 = 0$

$1 \div 1 = 1$

Example :-

$$
\begin{array}{r|l}
100 & 1111000 \quad (1110 \\
\hline
 & 100\downarrow \\
 & \hline
 & 111 \\
 & 100\downarrow \\
 & \hline
 & 110 \\
 & 100\downarrow \\
 & \hline
 & 100 \\
 & 100\downarrow \\
 & \hline
 & 000 \\
 & 000 \\
 & \hline
 & \times
\end{array}
$$

1110 is Ans.

⇒ Signed Binary Numbers and Binary Arithmetic

In binary number system, everything in the world is to be represented by a sequence of two numbers of the binary number system that is 0 and 1.

So, the +ve and -ve signs are also represented by 0 and 1. There are two methods of expressing the +ve and -ve numbers.

1. Signed binary numbers.
2. Unsigned binary numbers.

Signed binary numbers.

In this system the MSB (Most significant bit) or the left most bit is used as a sign bit.

For +ve number, the MSB is 0, and for -ve number, the MSB is 1.

So, $+6 = 00000110$

$-6 = 10000110$

S	M_6	M_5	M_4	M_3	M_2	M_1	M_0

MSB LSB

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Unsigned Number.

The natural binary number system where all the bits together give the magnitude of the number is called unsigned number. In this system, the number is always considered positive

M_7	M_6	M_5	M_4	M_3	M_2	M_1	M_0

MSB LSB

Complement of a number

The process of binary subtraction can be viewed as the addition of negative number.

For finding the -ve representation of a binary number, we must know 1's and 2's complement

1's complement of a binary number

The 1's complement of a binary number is found by subtracting each bit from 1

or

It can be found by changing each 0 to 1 and each 1 to 0.

If the given number is +ve then its 1's complement is -ve number with the same magnitude.

Binary Number	1's complement
10101	01010
1111100	0000011

Subtraction by using 1's complement of a number.

Subtracting by 1's complement when subtraction is of small number from a larger number. :-

Step 1 - Determine the 1's complement of a smaller number

Step 2 - Add the 1's complement to the larger number

Step 3 - Remove the carry and add the carry to the result.

Ex → Subtract the number $(10011)_2$ from $(11001)_2$ by using 1's complement

Ans Step 1:- Find the 1's complement of smaller number $(10011)_2$

$= 01100$

Step 2 :- Add 1's complement to the larger number

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$$\begin{array}{r} (1) \\ 01100 \\ +\ 11001 \\ \hline \text{Carry} \leftarrow 1\ 00101 \end{array}$$

Step 3 - Remove the carry and add carry in result of step 2

$$\begin{array}{r} \overset{(1)}{0010}1 \\ +\ 1 \\ \hline 00110 \end{array} \quad \text{Ans}$$

⇒ Method for subtracting a larger number from smaller number using 1's complement

Ex - Subtract $(1101)_2$ from $(1001)_2$

Step 1 :-

Find 1's complement of the larger number $(1101)_2$

$$= 0010$$

Step 2 - Add the 1's complement of the larger number to the small number

$$\begin{array}{r} 0010 \\ 1001 \\ \hline 1011 \end{array}$$

Step 3 :- As there is no carry, so do 1's complement of step 2 and assign opposite sign.

$$(-0100)_2$$

If there is carry in step 2, then remove the carry and add 1 to step 2 result to get the final result.

2's complement of a binary number.

2's complement of a binary number can obtained by adding 1 to the 1's complement.

Binary Number	1's complement	2's complement
10001	01110	01111
101010	010101	010110

Subtraction by using 2's complement

Ex Subtract $(10011)_2$ from $(11001)_2$ by using 2's complement method for subtraction.

Step 1, Find the 2's complement of the smaller number

$(10011)_2$

1's complement of the number = 01100

$+1$

2's complement 01101

Step 2 - Add the result of step 1 to the larger numbers

```
  01101
  11001
-------
  00110
```

Final carry. (1)

Step 3 - Discard the carry from the result of step 2

We get $(00110)_2$ Ans

For subtracting a larger number from a smaller number using the 2's complement method.

Ex → Subtract $(1101)_2$ from $(1001)_2$ by using 2's complement method of subtraction

Step 1 → Determine the 2's complement of larger number.

```
    1101
1's complement → 0010
                  +1
              -------
2's complement = 0011
```

Step 2 - Add 2's complement of larger number to the smaller number.

$$\begin{array}{r} (1)(1) \\ 0011 \\ +\ 1001 \\ \hline 1100 \end{array}$$

Step 3 → If there is no carry in MSB, then the result is in 2's complement form and negative

So, take the 2's complement of the result and change the sign

→ from 2's complement of number to normal form.

$$\begin{array}{r} 1100 \\ -1 \\ \hline 1011 \end{array}$$

$$\downarrow$$

$$(0100)$$

So solution of question is

$$-(0100)_2$$